空間均衡(外溢)效果
Spatial Equilibrium (Spillover) Effects

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  - Mathematical Relationships and Implications
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Spatial Lag Model

$$y = \rho W y + X\beta + \varepsilon$$

- Using Maximum Likelihood Estimation (MLE) to estimate rho (ρ) and beta (β).

TABLE 2.3. *MLE estimates of the spatially lagged y model.*

	$\hat{\beta}$	SE($\hat{\beta}$)	z-value
Intercept	-6.20	2.08	-2.98
Ln GDP per capita	0.99	0.28	3.59
ρ	0.56	0.08	7.43
N = 158			
Log likelihood (df=4) = -491.10			

Equilibrium (Spillover) Effects in Spatial Lag Model

$$y = \mathbf{X}\beta + \rho\mathbf{W}y + \epsilon.$$

$$\Rightarrow \quad (\mathbf{I} - \rho\mathbf{W})y = \mathbf{X}\beta + \epsilon.$$

spatial multiplier

$$\Rightarrow \quad E(y) = \boxed{(\mathbf{I} - \rho\mathbf{W})^{-1}}\mathbf{X}\beta.$$

This multiplier tells us how much of the change in x_i will "spill over" onto other states j and in turn affect y_i through the impact of y in the spatial lag.

Leontief Expansion

$$y = \rho W y + X\beta + \varepsilon \quad (3)$$

where ρ is the spatial autoregressive parameter with $|\rho| < 1$, W is an $n \times n$ row-standardized spatial weights matrix that represents the neighbor structure with spatial lag Wy as a weighted average of neighboring values, and the other variables are as in Eq. (1). After some manipulation, the reduced form of the spatial lag model can be expressed

$$y = (I - \rho W)^{-1} X\beta + (I - \rho W)^{-1} \varepsilon \quad (4)$$

where the "Leontief Inverse" $(I - \rho W)^{-1}$ links the dependent variable y to all the x_i in the system through a *spatial multiplier*. Note that expanding the "Leontief Inverse" matrix leads to an expanded form given that $|\rho| < 1$ and w_{ij}, the element of W, is less than 1 for row-standardized spatial weights:

$$(I - \rho W)^{-1} = I + \rho W + \rho^2 W^2 + \ldots \quad (5)$$

Leontief Expansion (cont'd)

$(I-\rho W)^{-1} = I + \rho W + \rho^2 W^2 + \rho^3 W^3 + \ldots$, then we have

$$y = (I + \rho W + \rho^2 W^2 + \rho^3 W^3 + \ldots)X\beta + (I + \rho W + \rho^2 W^2 + \rho^3 W^3 + \ldots)\varepsilon$$

$$= X\beta + \boxed{\rho W} X\beta + \boxed{\rho^2 W^2} X\beta + \ldots + \varepsilon + \rho W \varepsilon + \rho^2 W^2 \varepsilon + \ldots$$

ripple effect or diffusion process

Measuring Spillover Effects

$$y = \boxed{\rho W y} + X\beta + \varepsilon$$

- To understand how one state's GDP per capita affects the expected value of democracy in other states

$$(\mathbf{I} - \rho \mathbf{W})^{-1} \beta \Delta x(i)$$

Measuring Spillover Effects

Equilibrium impacts of log GDP per capita (X) for Russia

Country	Impact
Russia	1.09
People's Republic of Korea	0.24
Japan	0.24
Mongolia	0.24
Finland	0.22
Estonia	0.21
Norway	0.20
Lithuania	0.20
Latvia	0.120
Armenia	0.18

Effects on predicted democracy (Y) if China had a POLITY score of 10

Country	impact
Taiwan	1.88
North Korea	1.88
Mongolia	1.88
Nepal	1.41
Bhutan	1.41
Pakistan	1.13
Laos	1.13
Kyrgyzstan	1.13
Bangladesh	1.13
Uzbekistan	0.94
Thailand	0.94
Myanmar/Burma	0.94
Tajikistan	0.80
India	0.80
Vietnam	0.80
Afghanistan	0.80
Kazakhstan	0.70
Russia	0.28

Estimation of Spatial Spillover Effects

	A	B	C	D	E	F	G	H	I	J	K
1	W	j	1	2	3	4	5	6	7	8	9
2	i	1	0	1	0	1	1	0	0	0	0
3		2	1	0	1	1	1	1	0	0	0
4		3	0	1	0	0	1	1	0	0	0
5		4	1	1	0	0	1	0	1	1	0
6		5	1	1	1	1	0	1	1	1	1
7		6	0	1	1	0	1	0	0	1	1
8		7	0	0	0	1	1	0	0	1	0
9		8	0	0	0	1	1	1	1	0	1
10		9	0	0	0	0	1	1	0	1	0
11											

Excel 函數應用

反矩陣：=MINVERSE(m)

{contrl+shift+enter} 求解

矩陣相乘：=MMULT(n, x)

{contrl+shift+enter} 求解

spatial multiplier

$$E(y) = \boxed{(\mathbf{I} - \rho \mathbf{W})^{-1}} \mathbf{X}\beta.$$

Step-by-Step Instructions (Using Excel)

W (binary)

W	j	1	2	3	4	5	6	7	8	9	#Neighbor
i	1	0	1	0	1	1	0	0	0	0	3
	2	1	0	1	1	1	1	0	0	0	5
	3	0	1	[illegible]	[illegible]	[illegible]	[illegible]	0	0	0	3
	4	1	1	[illegible]	[illegible]	[illegible]	[illegible]	1	1	0	5
	5	1	1	[illegible]	[illegible]	[illegible]	[illegible]	1	1	1	8
	6	0	1	[illegible]	[illegible]	[illegible]	[illegible]	0	1	1	5
	7	0	0	0	1	1	0	0	1	0	3
	8	0	0	0	1	1	1	1	0	1	5
	9	0	0	0	0	1	1	0	1	0	3

W (row-standardized)

WN	j	1	2	3	4	5	6	7	8	9
i	1	0.000	0.333	0.000	0.333	0.333	0.000	0.000	0.000	0.000
	2	0.200	0.000	0.200	0.200	0.200	0.200	0.000	0.000	0.000
	3	0.000	0.333	0.000	0.000	0.333	0.333	0.000	0.000	0.000
	4	0.20	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.000
	5	0.12	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.125
	6	0.00	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.200
	7	0.000	0.000	0.000	0.333	0.333	0.000	0.000	0.333	0.000
	8	0.000	0.000	0.000	0.200	0.200	0.200	0.200	0.000	0.200
	9	0.000	0.000	0.000	0.000	0.333	0.333	0.000	0.333	0.000

RHO			0.7

Step-by-Step Instructions (cont'd)

I

I	j	1	2	3	4	5	6	7	8	9
i	1	1	0	0	0	0	0	0	0	0
	2	0	1	0	0	0	0	0	0	0
	3	0	0		0	0	0	0	0	0
	4	0	0		1	0	0	0	0	0
	5	0	0		0	1	0	0	0	0
	6	0	0		0	0	1	0	0	0
	7	0	0		0	0	0	1	0	0
	8	0	0	0	0	0	0	0	1	0
	9	0	0	0	0	0	0	0	0	1

ρW

j	1	2	3	4	5	6	7	8	9
1	0.000	0.233	0.000	0.233	0.233	0.000	0.000	0.000	0.000
2	0.140	0.000	0.140	0.140	0.140	0.140	0.000	0.000	0.000
3	0.000	0.23				:33	0.000	0.000	0.000
4	0.140	0.14				)00	0.140	0.140	0.000
5	0.088	0.08				)88	0.088	0.088	0.088
6	0.000	0.14				)00	0.000	0.140	0.140
7	0.000	0.00				)00	0.000	0.233	0.000
8	0.000	0.000	0.000	0.140	0.140	0.140	0.140	0.000	0.140
9	0.000	0.000	0.000	0.000	0.233	0.233	0.000	0.233	0.000

$I-\rho W$

(I-RO*WN)	j	1	2	3	4	5	6	7	8	9
i	1	1.000	-0.233	0.000	-0.233	-0.233	0.000	0.000	0.000	0.000
	2	-0.140	1.000	-0.140	-0.140	-0.140	-0.140	0.000	0.000	0.000
	3	0.000	-0.233				33	0.000	0.000	0.000
	4	-0.140	-0.140				00	-0.140	-0.140	0.000
	5	-0.088	-0.088				88	-0.088	-0.088	-0.088
	6	0.000	-0.140				00	0.000	-0.140	-0.140
	7	0.000	0.000				00	1.000	-0.233	0.000
	8	0.000	0.000	0.000	-0.140	-0.140	-0.140	-0.140	1.000	-0.140
	9	0.000	0.000	0.000	0.000	-0.233	-0.233	0.000	-0.233	1.000

$(I-\rho W)^{-1}$

SPATIAL MULTIPLIER (I-RO*WN)^-1	j	1	2	3	4	5	6	7	8	9
i	1	1.16987	0.43938	0.134924	0.439377	0.53541	0.1897	0.1349	0.1897	0.1
	2	0.26363	1.25707	0.263626	0.331496	0.47116	0.3315	0.1138	0.1872	0.1138
	3	0.13492	0.43938	1.						.1349
	4	0.26363	0.3315	0						.1138
	5	0.20078	0.29448	0						.2008
	6	0.11384	0.3315	0.						.2636

預期結果 (Impulse of X or Y)

Impulse of X:

Effect= $(I-\rho W)^{-1}X$

Impulse of Y:

Effect = ρWy

i	*X_IMPULSE*	*EFFECT*	*Y_IMPULSE*	*EFFECT*	*# NEIGHB*		*RHO*
1	1	1.169874	1	0.000	3		0.7
2	0	0.263626	0	0.200	5		
3	0	0.134924	0	0.000	3		
4	0	0.263626	0	0.200	5		
5	0	0.20078	0	0.125	8		
6	0	0.11384	0	0.000	5		
7	0	0.134924	0	0.000	3		
8	0	0.11384	0	0.000	5		
9	0	0.099974	0	0.000	3		
SUM		2.495411		0.525			

複習：Spatial Lag Model

$$y = \rho W y + X\beta + \varepsilon$$

y: democracy score

x: GDP per capita

Spatial Spillover Effect

某國家的x變化一個單位產生的 y(民主化)擴散效果

Equilibrium impacts of log GDP per capita (X) for ***Russia***

Country	Impact
Russia	1.09
People's Republic of Korea	0.24
Japan	0.24
Mongolia	0.24
Finland	0.22
Estonia	0.21
Norway	0.20
Lithuania	0.20
Latvia	0.120
Armenia	0.18

某國家的y變化所產生的 y(民主化)擴散效果

Effects on predicted democracy (Y) if ***China*** had a POLITY score of 10

Country	impact
Taiwan	1.88
North Korea	1.88
Mongolia	1.88
Nepal	1.41
Bhutan	1.41
Pakistan	1.13
Laos	1.13
Kyrgyzstan	1.13
Bangladesh	1.13
Uzbekistan	0.94
Thailand	0.94
Myanmar/Burma	0.94
Tajikistan	0.80
India	0.80
Vietnam	0.80
Afghanistan	0.80
Kazakhstan	0.70
Russia	0.28

複習：Measuring Spatial Spillover Effect

將x變化一個單位，對y的擴散效果

$$y = \mathbf{X}\beta + \rho \mathbf{W} y + \epsilon.$$

$$\Rightarrow (\mathbf{I} - \rho \mathbf{W}) y = \mathbf{X}\beta + \epsilon.$$

spatial multiplier

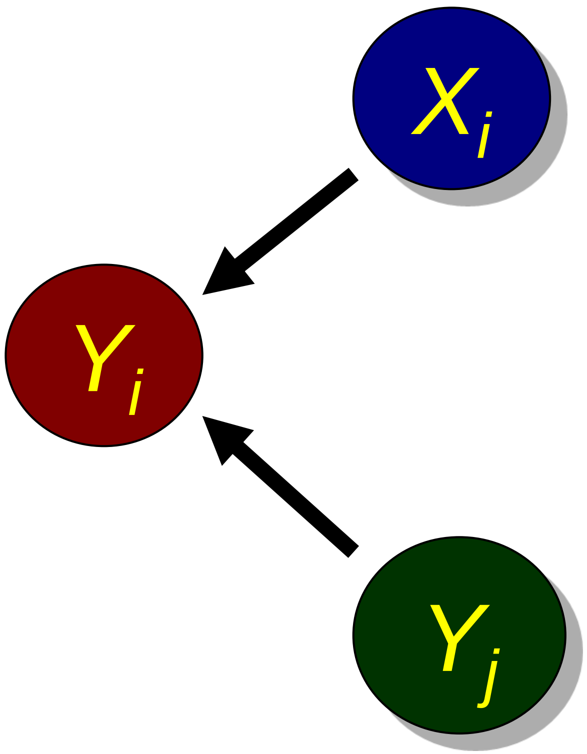
$$\Rightarrow E(y) = (\mathbf{I} - \rho \mathbf{W})^{-1} \mathbf{X}\beta.$$

將y變化一個單位，對y的擴散效果

Holding X **and the other parameters constant**

$$y = \rho W y + X\beta + \varepsilon$$

複習：Excel實作



j	X_IMPULS	EFFECT	Y_IMPULS	EFFECT	# NEIGHB		RHO
1	1	1.16987	1	1.000	3		0.7
2	0	0.26363	0	0.140	5		
3	0	0.13492	0	0.000	3		
4	0	0.26363	0	0.140	5		
5	0	0.20078	0	0.088	8		
6	0	0.11384	0	0.000	5		
7	0	0.13492	0	0.000	3		
8	0	0.11384	0	0.000	5		
9	0	0.09997	0	0.000	3		
SUM		**2.49541**		**1.3675**			

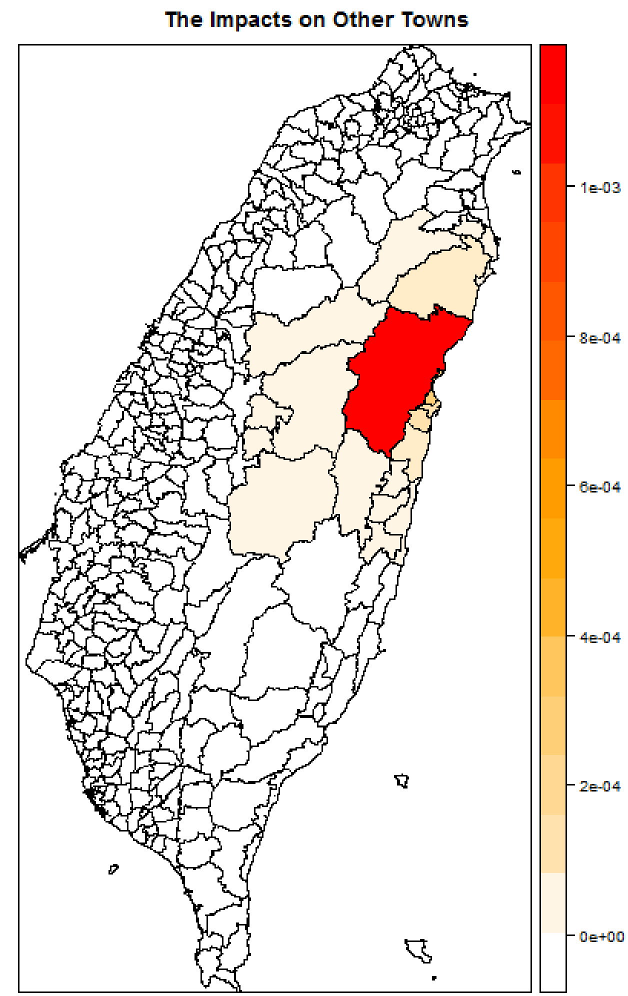
j	X_IMPULS	EFFECT	Y_IMPULS	EFFECT	# NEIGHB		RHO
1	1	1.74418	1	1.233	3		0.7
2	1	1.78432	1	1.280	5		
3	1	1.74418	1	1.233	3		
4	0	0.70896	0	0.280	5		
5	0	0.69604	0	0.263	8		
6	0	0.70896	0	0.280	5		
7	0	0.42463	0	0.000	3		
8	0	0.41485	0	0.000	5		
9	0	0.42463	0	0.000	3		
SUM		**8.65076**		**4.56917**			

R Lab: The impact of change in a town (秀林鄉) on other towns

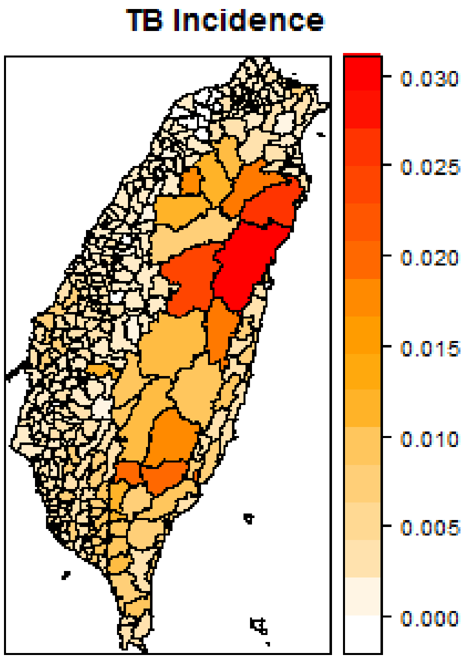
秀林鄉的 X (原民比例) 增加 1 單位，

Y (TB發生率) 的增加量

	TBData.FULLNAME	rus.est
301	花蓮縣秀林鄉	0.00111
294	花蓮縣新城鄉	0.00024
291	花蓮縣花蓮市	0.00019
295	花蓮縣吉安鄉	0.00017
296	花蓮縣壽豐鄉	0.00010
41	宜蘭縣南澳鄉	0.00009
145	南投縣仁愛鄉	0.00007
302	花蓮縣萬榮鄉	0.00006
106	台中縣和平鄉	0.00005
292	花蓮縣鳳林鎮	0.00002



R code: Loading Shapefiles




```
rm(list=ls())
setwd("D:/R_Labs/SA2018")
library(rgdal)
library (spdep)

# Load Shapefiles
TWN.TB <- readOGR(dsn = "SHP", layer = "Taiwan_TB", encoding="utf-8")

head(TWN.TB@data)
lm.palette <- colorRampPalette(c("white","orange", "red"), space = "rgb")
spplot(TWN.TB, zcol="TBINCI", col.regions=lm.palette(20), main="TB Incidence")
```

R code: Neighbors and Weighting Matrix

```
# Neighbors: Construct neighbors list
TWN_nbq<-poly2nb(TWN.TB) #QUEEN = TRUE
summary(TWN_nbq)

# Neighborhood Matrix
TWN_nbq_w.mat <-nb2mat(TWN_nbq, style="W", zero.policy=T) # row-standardized
TWN_nbq_w2.mat <-nb2mat(TWN_nbq, style="B", zero.policy=T) # binary

# Row-standardized weights matrix (list)
TWN_nbq_w<- nb2listw(TWN_nbq, zero.policy=T)
# Binary matrix (list)
TWN_nbq_wb2<-nb2listw(TWN_nbq, style="B", zero.policy=T)
```

R code: OLS Model

```
# OLS Regression
TBData<-TWN.TB@data
TBINCI<-TWN.TB@data$TBINCI # TB發生率
ABOR_P<-TWN.TB@data$ABOR_P # 原住民比例
INCOME<-TWN.TB@data$INCOME # 平均家戶收入

TB.lm<- lm(TBINCI ~ ABOR_P + INCOME, data=TBData); summary(TB.lm)

# Global Moran's I for LM regression residuals
TB.moran0 <-  lm.morantest(TB.lm, TWN_nbq_w, zero.policy=T); TB.moran0

#Lagrange Multiplier Test Statistics for Spatial Autocorrelation
TB.lagrange <- lm.LMtests(TB.lm,TWN_nbq_w,test=c("LMerr","RLMerr","LMlag","RLMlag","SARMA"), zero.policy=T)
summary(TB.lagrange)

# MLE of the Spatial Lag Model
TB.lag <- lagsarlm(TBINCI ~ ABOR_P + INCOME, data=TBData, TWN_nbq_w, zero.policy=T); summary(TB.lag)
```

```
> TB.lm<- lm(TBINCI ~ ABOR_P + INCOME, data=TBData); summary(TB.lm)

Call:
lm(formula = TBINCI ~ ABOR_P + INCOME, data = TBData)

Residuals:
       Min         1Q     Median         3Q        Max 
-0.0065203 -0.0009545 -0.0000505  0.0009511  0.0155064 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept)  4.852e-03  3.310e-04  14.657  < 2e-16 ***
ABOR_P       1.273e-02  7.155e-04  17.797  < 2e-16 ***
INCOME      -6.510e-06  2.135e-06  -3.049  0.00247 ** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

R code: Results of Spatial Lag Model

```
Call:lagsarlm(formula = TBINCI ~ ABOR_P + INCOME, data = TBData, listw = TWN_nbq_w,
    zero.policy = T)

Residuals:
       Min         1Q      Median          3Q        Max
-8.0582e-03 -7.2283e-04 -6.3813e-06  8.4425e-04  1.4849e-02

Type: lag
Regions with no neighbours included:
 262 284 289
Coefficients: (numerical Hessian approximate standard errors)
               Estimate  Std. Error z value  Pr(>|z|)
(Intercept)  2.8197e-03  4.3929e-04  6.4188 1.374e-10
ABOR_P       1.0405e-02  7.5813e-04 13.7247 < 2.2e-16
INCOME      -3.1862e-06  2.0478e-06 -1.5559    0.1197

Rho: 0.34805, LR test value: 37.499, p-value: 9.1474e-10
Approximate (numerical Hessian) standard error: 0.053804
    z-value: 6.4689, p-value: 9.8691e-11
Wald statistic: 41.847, p-value: 9.8691e-11

Log likelihood: 1640.718 for lag model
ML residual variance (sigma squared): 5.1089e-06, (sigma: 0.0022603)
Number of observations: 352
Number of parameters estimated: 5
AIC: -3271.4, (AIC for lm: -3235.9)
```

R code: Estimation of Spatial Spillover Effects

```
# Modeling Spatial Equilibrium Effect
TB.lag2 <- lagsarlm(TBINCI ~ ABOR_P, data=TBData, TWN_nbq_w, zero.policy=T); summary(TB.lag2)
rho<-coef(TB.lag2)[1]
beta<-coef(TB.lag2)[3]
```

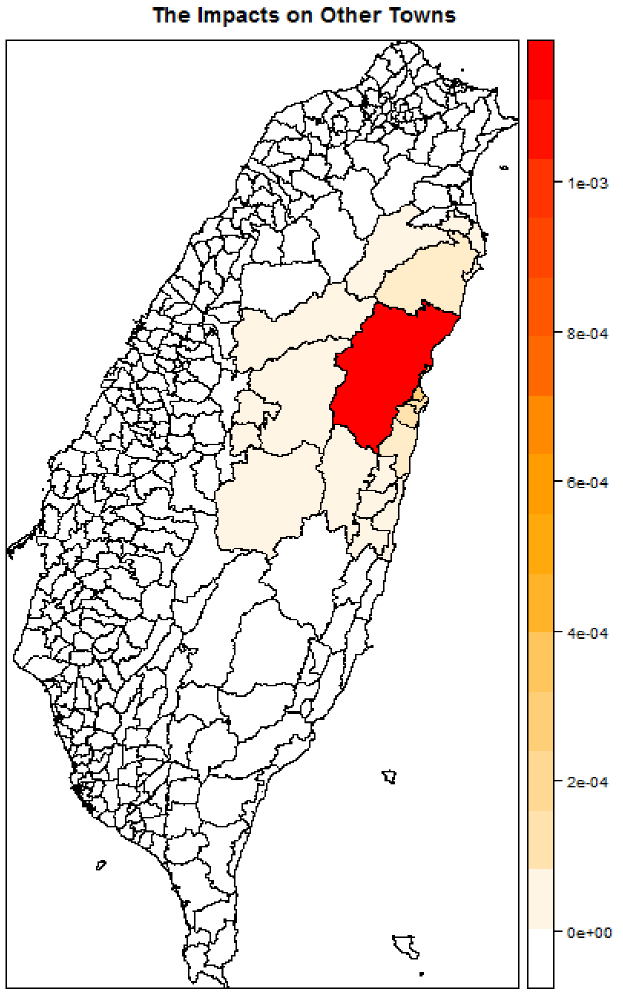
```
Call:lagsarlm(formula = TBINCI ~ ABOR_P, data = TBData, listw = TWN_nbq_w,     zero.policy = T)

Residuals:
       Min         1Q     Median         3Q        Max
-8.3573e-03 -7.3359e-04  2.5877e-05  8.7969e-04  1.4728e-02

Type: lag
Regions with no neighbours included:
 262 284 289
Coefficients: (numerical Hessian approximate standard errors)
             Estimate Std. Error z value  Pr(>|z|)
(Intercept) 0.0022801  0.0002673  8.5299 < 2.2e-16
ABOR_P      0.0106192  0.0007466 14.2235 < 2.2e-16

Rho: 0.36831, LR test value: 44.353, p-value: 2.7419e-11
```

R code: The impact of change in a town (秀林鄉) on other towns

```
# example of impact on other townships (observation No.301)
cvec <- rep(0,dim(TBData)[1])
cvec[301] <- 0.1 # 花蓮縣秀林鄉

# Store estimates for impact of change in one town in rus.est
eye <- matrix(0,nrow=dim(TBData)[1],ncol=dim(TBData)[1])
diag(eye) <- 1
rus.est <- solve(eye - rho * TWN_nbq_w.mat) %*% cvec * beta


# Find ten highest values of rus.est vector
rus.est <- round(rus.est,6)
rus.est <- data.frame(TBData$FULLNAME,rus.est)
rus.est[rev(order(rus.est$rus.est)),][1:10,]

TWN.TB$rus.est <-rus.est[,2]
spplot(TWN.TB, zcol="rus.est", col.regions=lm.palette(20), main="TB Spillover Effects")
```

R code: The impact of change in a town (秀林鄉) on other towns

```
    TBData.FULLNAME rus.est
301    花蓮縣秀林鄉 0.00111
294    花蓮縣新城鄉 0.00024
291    花蓮縣花蓮市 0.00019
295    花蓮縣吉安鄉 0.00017
296    花蓮縣壽豐鄉 0.00010
41     宜蘭縣南澳鄉 0.00009
145    南投縣仁愛鄉 0.00007
302    花蓮縣萬榮鄉 0.00006
106    台中縣和平鄉 0.00005
292    花蓮縣鳳林鎮 0.00002
```



Example: Tuberculosis (TB) Diffusion

OPEN ACCESS Freely available online

PLOS | ONE

Spatial Dependency of Tuberculosis Incidence in Taiwan

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Abstract

Tuberculosis (TB) disease can be caused by either recent transmission from infectious patients or reactivation of remote latent infection. Spatial dependency (correlation between nearby geographic areas) in tuberculosis incidence is a signature for chains of recent transmission with geographic diffusion. To understand the contribution of recent transmission in the TB endemic in Taiwan, where reactivation has been assumed to be the predominant mode of pathogenesis, we used spatial regression analysis to examine whether there was spatial dependency between the TB incidence in each township and in its neighbors. A total of 90,661 TB cases from 349 townships in 2003–2008 were included in this analysis. After adjusting for the effects of confounding socioeconomic variables, including the percentages of aboriginals and average household income, the results show that the spatial lag parameter remains positively significant (0.43, $p<0.001$), which indicates that the TB incidences of neighboring townships had an effect on the TB incidence in each township. Townships with substantial spatial spillover effects were mainly located in the northern, western and eastern parts of Taiwan. Spatial dependency implies that recent transmission plays a significant role in the pathogenesis of TB in Taiwan. Therefore, in addition to the current focus on improving the cure rate under directly observed therapy programs, more resource need to be allocated to active case finding in order to break the chain of transmission.

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Spatial Models

- OLS Model
  - $y = \beta X + \varepsilon$
- Spatial Lag Model
  - $y = \rho W y + \beta X + \varepsilon$
- Spatial-temporal Lag Model
  - $y_{(t)} = \rho W y_{(t-1)} + \beta X + \varepsilon$

Results: Model Comparisons

Table 3. Multiple regression analyses: ordinary least square (OLS) model, spatial lag model, and spatial time lag model.

Variable	OLS model^	Spatial Lag model†	Spatial-Time Lag model††
ABOR_P	1.38***	1.19***	1.15***
INCOME2	−0.15***	−0.08	−0.07
INCOME3	−0.34***	−0.21***	−0.22***
Spatial Lag (Wy)	–	0.43***	–
Spatial Time Lag (Wy_{t-1})	–	–	64.63**
Adjusted R^2	0.53	–	0.42
Log likelihood	−97.04	−78.52	−137.21
AIC	202.08	167.05	284.42

*p<0.05
**p<0.01
***p<0.001
^†Dependent variable: ln (TB_INCI)
††Dependent variable: ln (TB_INCI_6).
See Table 1 for variables abbreviation AIC: Akaike's information criterion.
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Results: Spatial Multiplier Effect

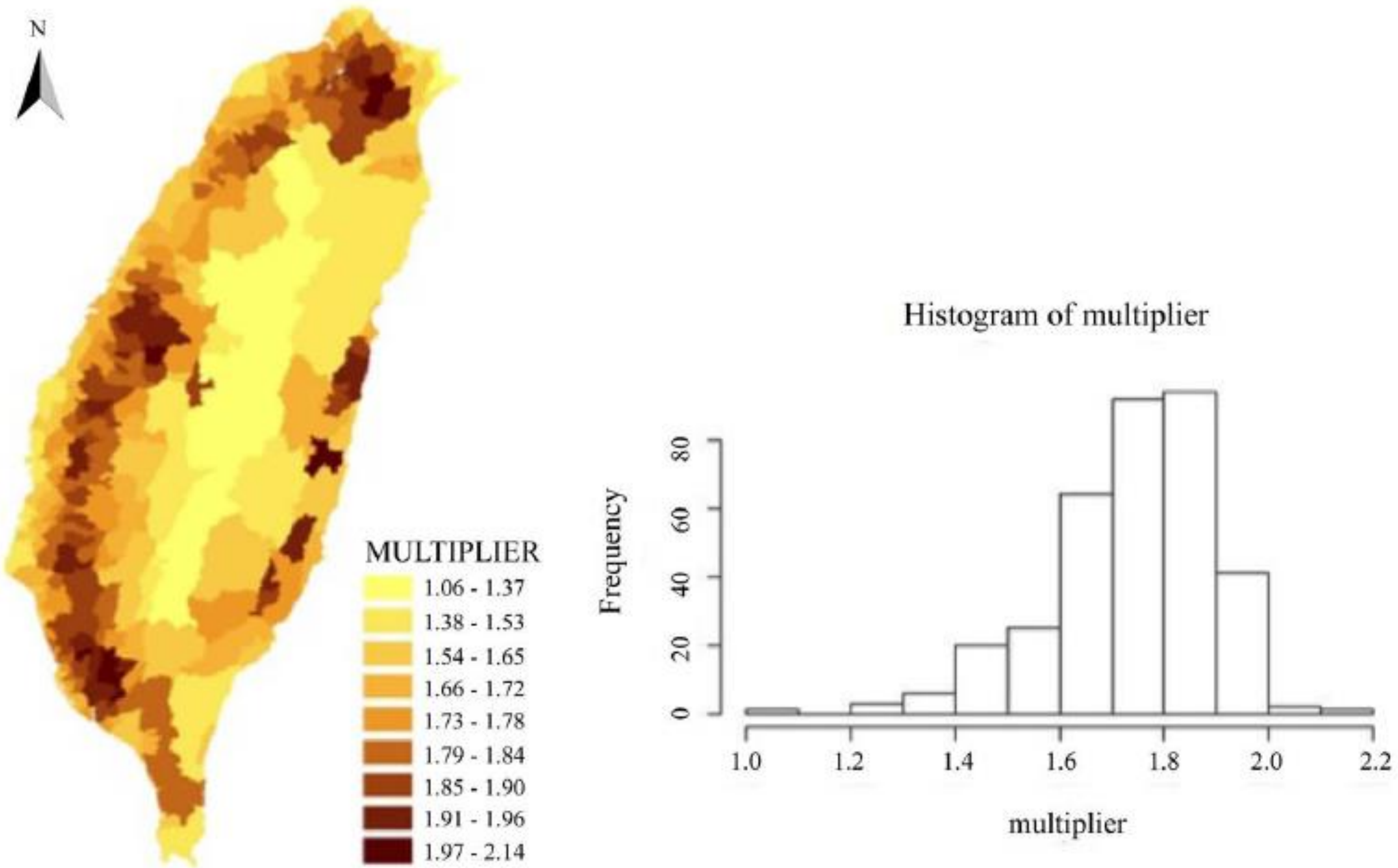

Figure 2. Spatial variations and the histogram of spatial multipliers.
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