半變異元分析

空間分析 2020.06.08

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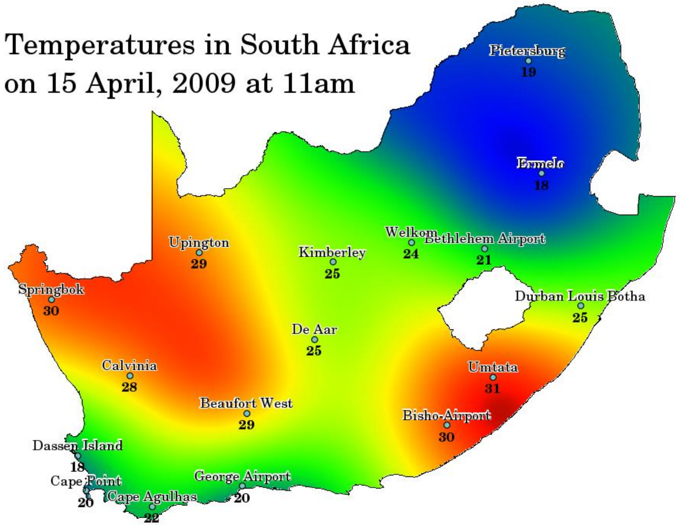
semi-variogram $\gamma(h)$

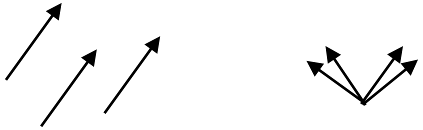
- The variogram is defined as the variance of the difference between field values at two locations (s_i, s_j) across realizations of the field.

$$2\gamma(s_i, s_j) = \mathrm{Var}(Z(s_i) - Z(s_j)) = \mathrm{E}\left[\left(Z(s_i) - Z(s_j)\right)^2\right]$$

field has constant mean $\mu(s_i) = \mu(s_j)$



- s_i - location (station)
- $Z(s_i)$ - value (temperature)



- stationary & isotropic

$$2\gamma(s_i, s_j) = 2\gamma(h) = \mathrm{Var}(Z(x+h) - Z(x)) = \mathrm{E}[(Z(x+h) - Z(x))^2] = \frac{1}{S}\int_S [Z(x+h) - Z(x)]^2 \, dA$$

$h = s_i - s_j$

semi-variogram & covariance function

semi-variogram - $\gamma(s_i, s_j) = \frac{1}{2} Var(Z(s_i) - Z(s_j))$

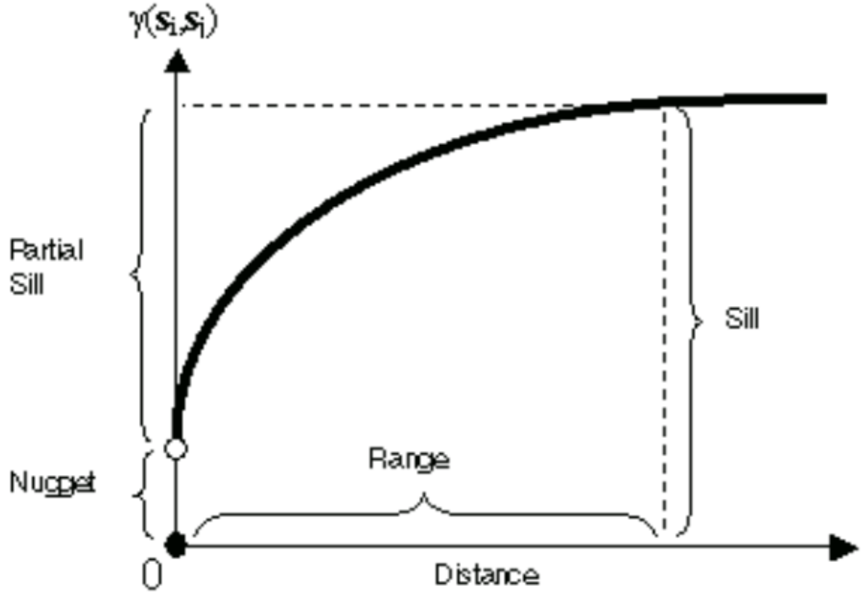
covariance function - $C(s_i, s_j) = Cov(Z(s_i), Z(s_j))$

$\Rightarrow \gamma(s_i, s_j) = \sigma_Z^2 - C(s_i, s_j)$

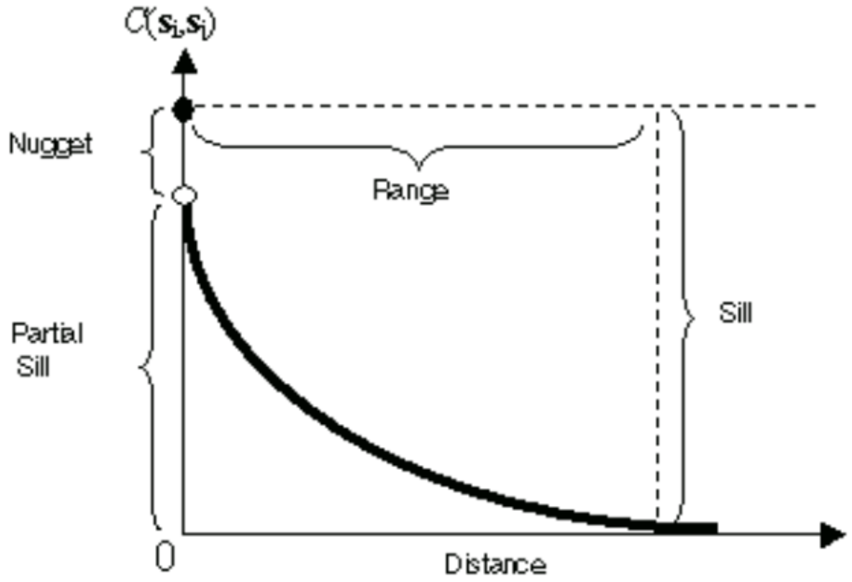
$$2\gamma(s_i, s_j) = Var(Z(s_i) - Z(s_j))$$
$$= Var(Z(s_i)) + Var(Z(s_j)) - 2\,Cov(Z(s_i), Z(s_j))$$
$$\uparrow$$

- $\mathrm{Var}(X + Y) = Var(X) + Var(Y) + 2\,Cov(X, Y)$
- $\mathrm{Var}(X - Y) = Var(X) + Var(Y) - 2\,Cov(X, Y)$

semi-variogram



covariance



$\gamma(s_i, s_j) = \text{sill} - C(s_i, s_j)$

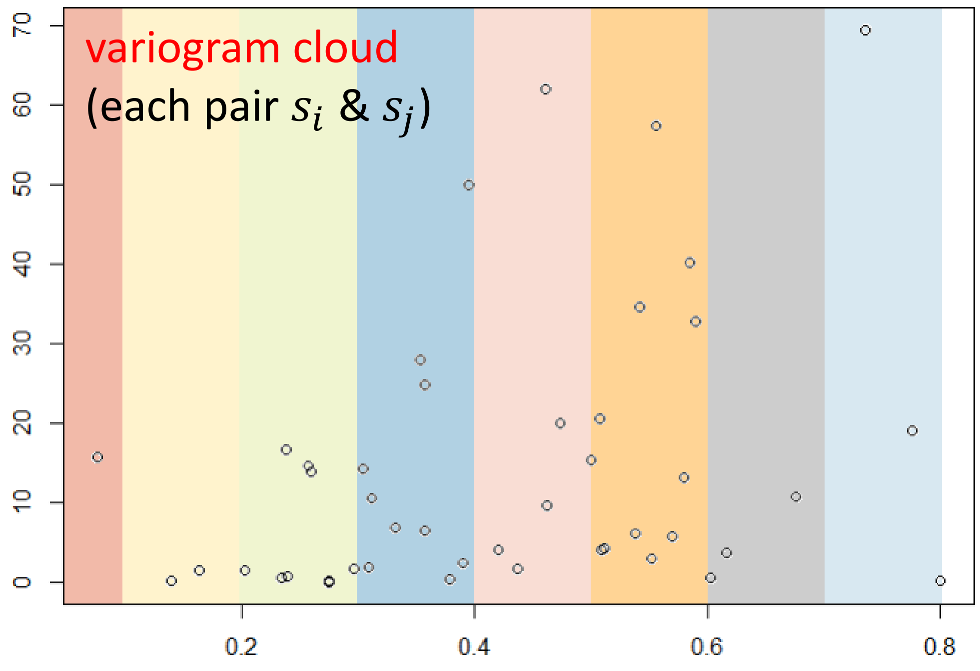
https://desktop.arcgis.com/en/arcmap/latest/extensions/geostatistical-analyst/semivariogram-and-covariance-functions.htm

empirical semi-variogram

$$\gamma(h) = \frac{1}{2S}\int_S [Z(x+h) - Z(x)]^2 \, dA$$

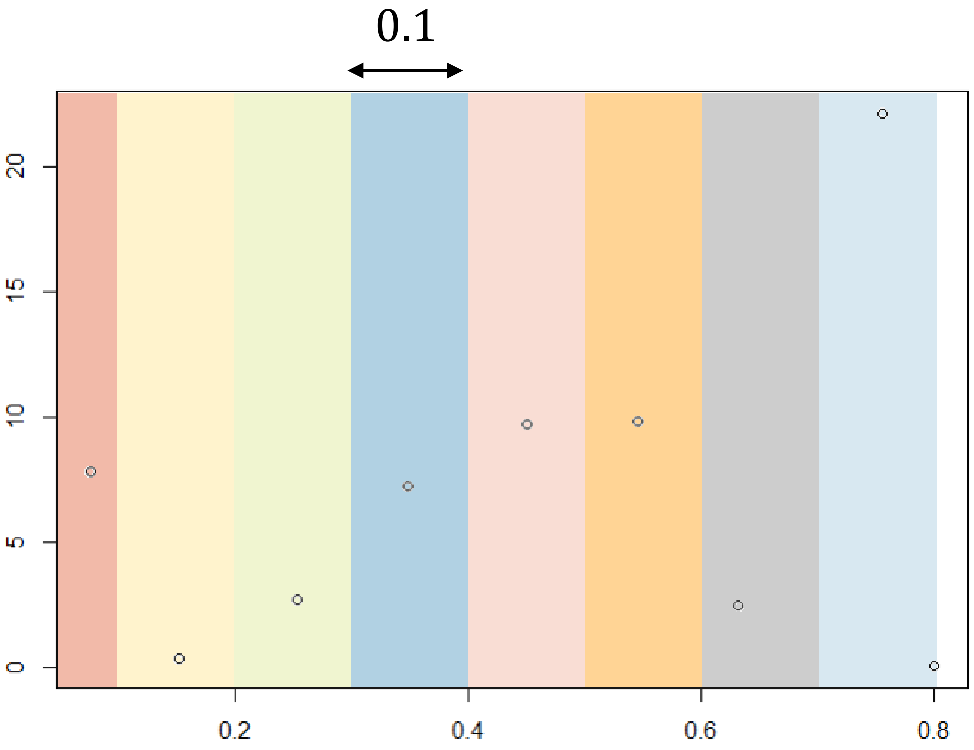
$$\hat{\gamma}(h) = \frac{1}{2n(h)}\sum_{i=1}^{n(h)}[Z(x+h) - Z(x)]^2 = \frac{1}{2n(h)}\sum_{i=1}^{n(h)}\left[Z(s_i) - Z(s_j)\right]^2, |s_i - s_j| \in h$$

$2\gamma(s_i, s_j) = \left(Z(s_i) - Z(s_j)\right)^2$



$|s_i - s_j|$

$\hat{\gamma}(h)$



h

R-code

- $|s_i - s_j|$

```
d=dist(SP@coords)
```

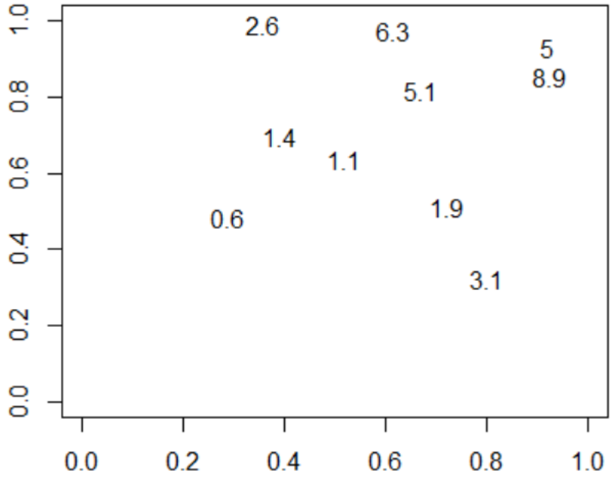
```
> d
        1      2      3      4      5      6      7      8      9
2  0.5097
3  0.3573 0.5082
4  0.6028 0.4374 0.3116
5  0.2605 0.5896 0.1641 0.4743
6  0.3900 0.2753 0.2388 0.2397 0.3531
7  0.2964 0.2351 0.3046 0.3795 0.3578 0.1401
8  0.5700 0.7756 0.2758 0.4622 0.3100 0.5005 0.5804
9  0.5850 0.7359 0.2569 0.3952 0.3317 0.4613 0.5558 0.0768
10 0.8004 0.5381 0.5118 0.2030 0.6755 0.4202 0.5526 0.6171 0.5425
```

- $[Z(s_i) - Z(s_j)]^2$

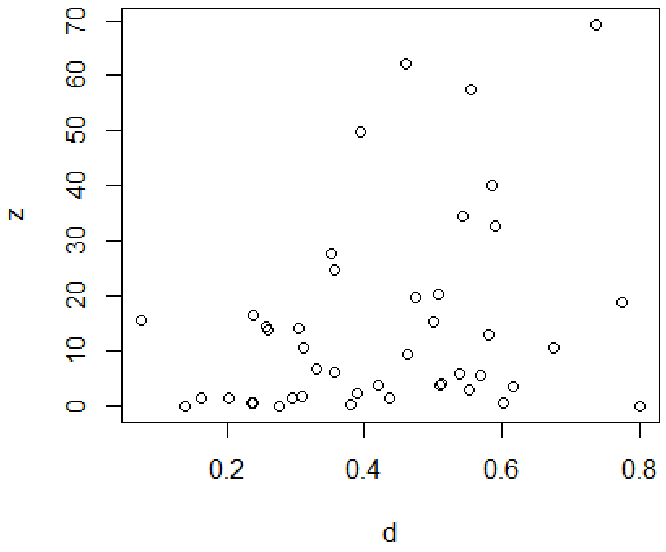
```
z=dist(SP$z)^2
```

```
> z
         1       2       3       4       5       6       7       8       9
2   3.9817
3   6.3783 20.4389
4   0.5324  1.6021 10.5964
5  13.9256 32.7999  1.4549 19.9040
6   2.3848  0.2035 16.5633  0.6636 27.8360
7   1.5454  0.5659 14.2030  0.2637 24.7492  0.0907
8   5.6113 19.0466  0.0246  9.6008  1.8574 15.3124 13.0464
9  40.1192 69.3788 14.5043 49.8953  6.7718 62.0669 57.4129 15.7224
10  0.2048  5.9923  4.2974  1.3976 10.7531  3.9872  2.8753  3.6723 34.5917
```

- data: SP (points)
- value: column z

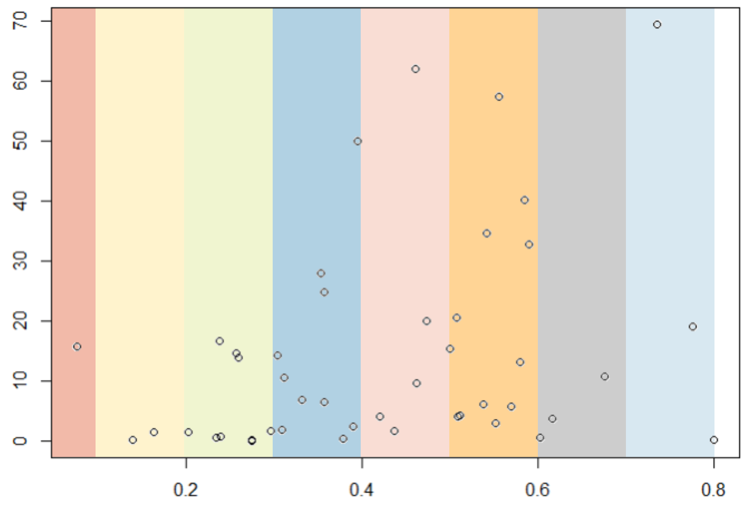


- plot(z~d)

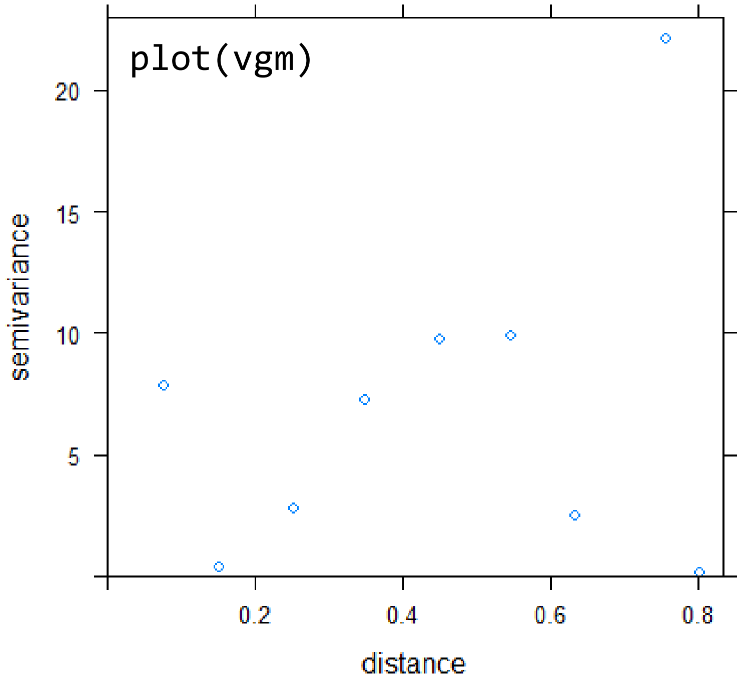


R-code

```
library(gstat)
vgm = variogram(SP$z~1,SP,cutoff=0.9,width=0.1)
```

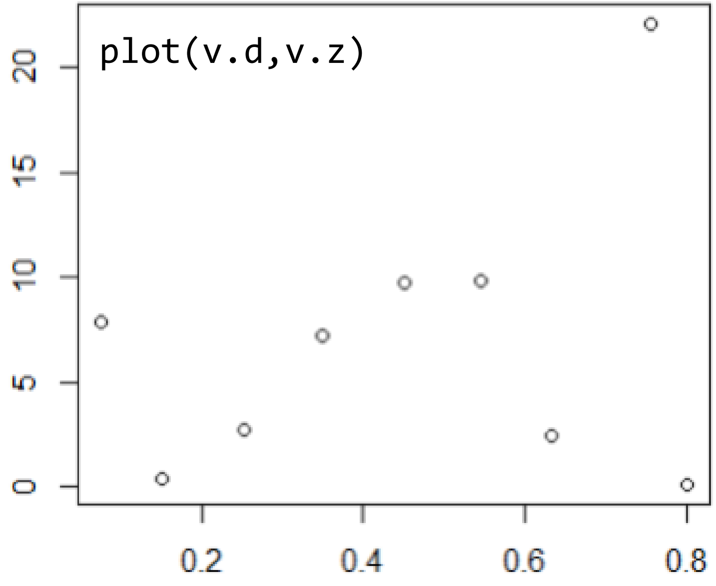



```
> vgm
  np   dist   gamma
1  1 0.0768   7.861
2  2 0.1521   0.386
3  9 0.2535   2.744
4 10 0.3491   7.247
5  5 0.4511   9.716
6 12 0.5453   9.853
7  3 0.6318   2.493
8  2 0.7558  22.106
9  1 0.8004   0.102
```

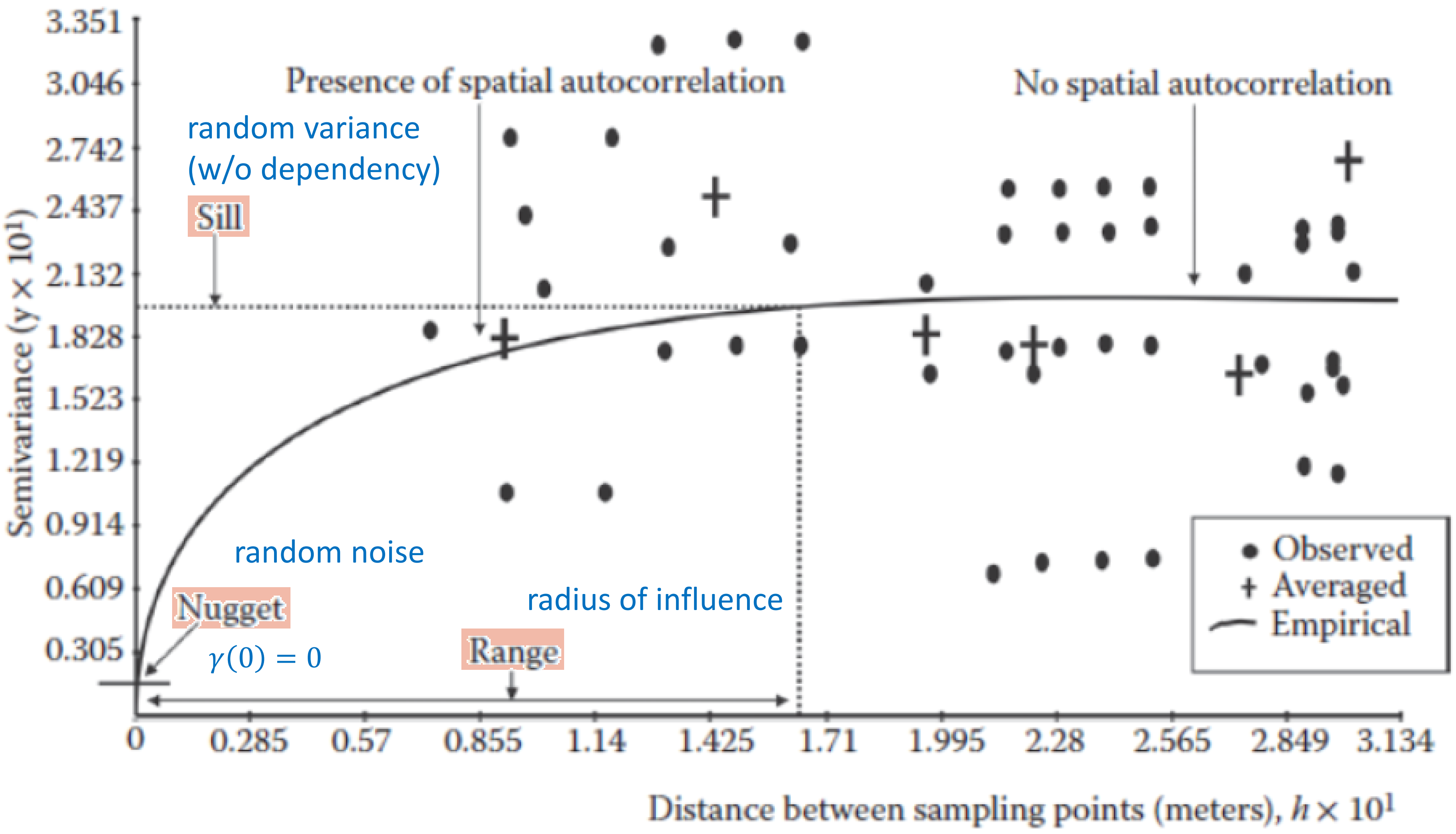



```
x=seq(0,cutoff,width)
x=seq(0,0.9,0.1)
v.d=v.z=c()
for(i in 1:(length(x)-1)){
  judge = d>x[i] & d<x[i+1]
  v.d[i]=mean(d[judge])
  v.z[i]=mean(z[judge])/2
}
```

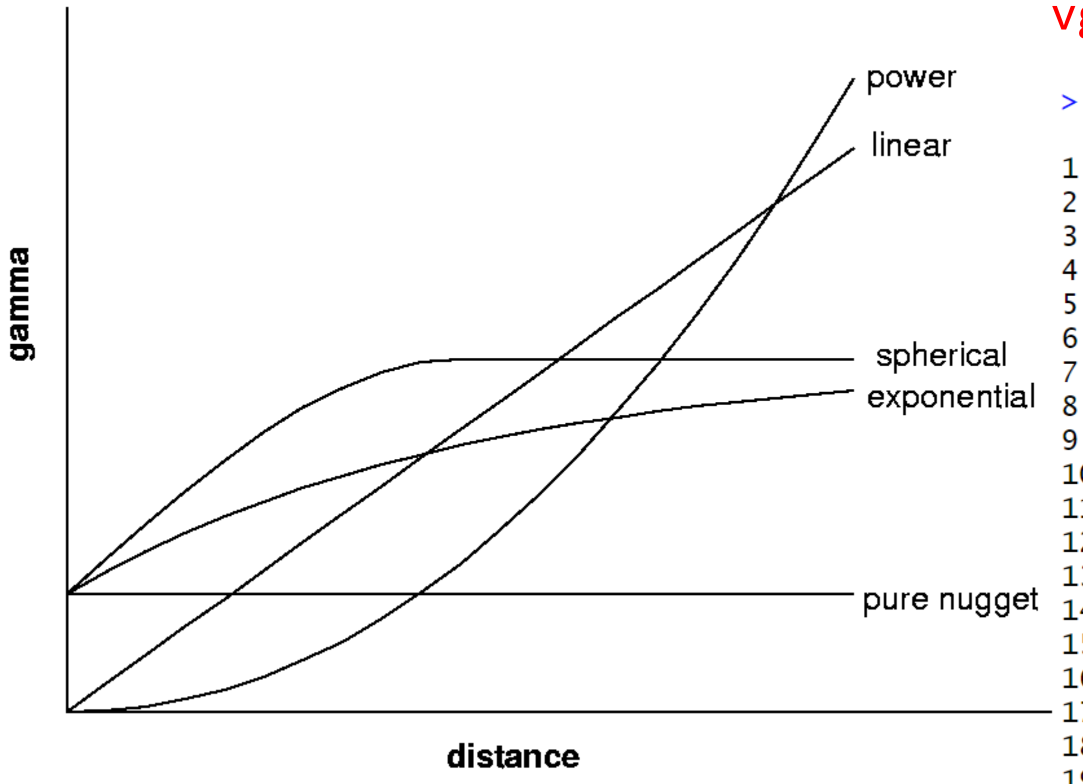
```
> data.frame(v.d,v.z)
     v.d     v.z
1 0.0768   7.861
2 0.1521   0.386
3 0.2535   2.744
4 0.3491   7.247
5 0.4511   9.716
6 0.5453   9.853
7 0.6318   2.493
8 0.7558  22.106
9 0.8004   0.102
```



variogram model



variogram model




```
fit.variogram(vgm, model = vgm(500, "Exp",30000,5))

vgm(sill, model, range, nugget)
```

```
> vgm()
   short                                        long
1    Nug                                 Nug (nugget)
2    Exp                            Exp (exponential)
3    Sph                              Sph (spherical)
4    Gau                               Gau (gaussian)
5    Exc        Exclass (Exponential class/stable)
6    Mat                                 Mat (Matern)
7    Ste Mat (Matern, M. Stein's parameterization)
8    Cir                               Cir (circular)
9    Lin                                 Lin (linear)
10   Bes                                 Bes (bessel)
11   Pen                          Pen (pentaspherical)
12   Per                               Per (periodic)
13   Wav                                   Wav (wave)
14   Hol                                   Hol (hole)
15   Log                            Log (logarithmic)
16   Pow                                  Pow (power)
17   Spl                                 Spl (spline)
18   Leg                               Leg (Legendre)
19   Err                     Err (Measurement error)
20   Int                              Int (Intercept)
```